Speed Planning for Autonomous Driving via Convex Optimization

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October 25, 2018

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Outline

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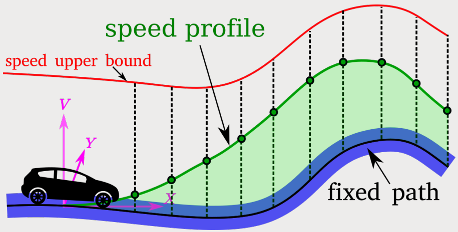
Motivation

Speed Planning for Autonomous Driving

Speed planning exists in two main motion planning frameworks

- **Coupled motion planning framework**
  - Explore the spatial-temporal space simultaneously using optimization techniques or search algorithms
- **Decoupled motion planning framework**
  - Plan a path first, then regulate the speed along the resulting path
  - Do speed planning directly along a fixed path (**the focus of this paper**)



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Motivation

Challenges from Various Scenarios

A speed planner should be able to

Requirements

- Exploit the full mobility capacity of cars
  to deal with emergencies
- Encourage smooth speed profiles
  for ride comfort , better tracking performance
- Pursue time efficiency
  e.g. drive on the limits to pursue high speeds, racing car
- Avoid static obstacles
  e.g. safe stop at certain point of the path
- Avoid dynamic obstacles
  e.g. moving vehicles, cyclists, pedestrians

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Motivation

Metrics and Requirements

Constraints for speed planning

Category	Constraint Name	Description	Property
Soft Constraints	Smoothness (S) Time Efficiency (TE)	continuity of speed, acceleration and jerk over the path time used by travelling along the path	performance performance
Hard Constraints	Friction Circle (FC) Time Window (TW) Boundary Condition (BC)	total force should be within the friction circle time window to reach a certain point on path speed at the end of the path	safety safety safety&performance

Remark

A safety-guaranteed speed planner should be able to generate a solution satisfying at least all the **hard constraints (safety)** in the Table.

A mature speed planner should cover all these constraints that include **soft and hard ones**.

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Motivation

State-of-the-art Methods

Capacity of different speed planning methods

Method	Name	S	TE	FC	TW	BC	Optimality	Safety	Mobility	Flexibility
Li *et al.* [1]	Trapezoid	✓	✗	✗	✗	✓	✗	low	low	low
Gu *et al.* [2, 3, 4]	Constraint-based	✓	✗	✗	✗	✓	✗	medium	medium	medium
Dakibay *et al.* [5]	Approximated	✗	✗	✓	✗	✓	✗	medium	high	low
Liu *et al.* [6]	SCFS	✓	✓✗	✗	✓	✓✗	local	medium	medium	medium
Lipp *et al.* [7]	MTSOS	✗	✓	✓	✗	✗	global	low	high	low

S: smoothness, **TE**: time efficiency, **FC**: friction circle, **TW**: time window, **BC**: boundary condition

Mobility How much mobility capacity of the vehicle the planner is able to leverage.

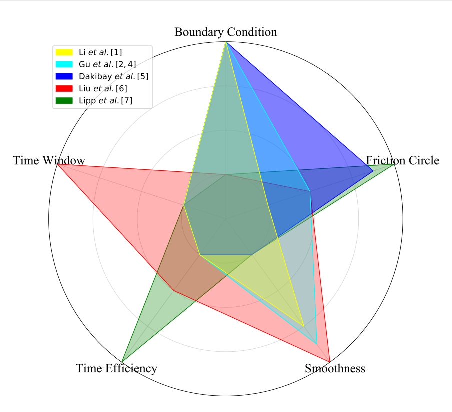
Optimality Whether the planner is able to identify an optimal solution in terms of its objective.

Flexibility How many type of scenarios the planner is able to handle by only adjusting parameters without changing underlying problem formulation or problem structures.

Safety Ability to stop in front of obstacles (BC) precisely, ability to deal with emergencies (FC), and ability to handle dynamic obstacles (TW).

Motivation

Limitations of existing methods



Limitations

- Not every aspect is covered (Completeness)
- Acceleration capacity is not fully exploited (Friction Circle)
- Dynamic obstacles are not handled reasonably well (Time Window)
- Smoothness is ignored by some of them (Ride Comfort, Tracking Performance)

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Motivation

Our Goals

Problem Definition

Assuming a curvature continuous path has been generated by a hierarchical motion planning framework, the speed planning is to find a

- **time-efficient**
- **safe**
- **smooth**

speed profile travelling along the fixed path with respect to both safety and performance constraints.

Our Goals

- employ a unified framework to deal with various driving scenarios
- cover safety, comfort, time efficiency and mobility
- prefer an elegant mathematical model instead of error-prone algorithms

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Methods – Preliminaries

- **Path Representation**

  General arc-length representation in Cartesian coordinate system

  $$r(s) = (x(s), y(s)), s \in [0, s_f]$$

  Any path can be easily converted to this form.
- **Associate speed profiles with paths** in the arc-length one dimension space by

  $$s = f(t)$$
- **Math Trick** according to Verscheure *et al.*[8]

  $$\alpha(s) = \ddot{f}, \quad \textbf{longitudinal acceleration}$$
  $$\beta(s) = \dot{f}^2, \quad \textbf{square of the longitudinal speed}$$

  The prime ′ and the dot · denote derivatives with respect to the arc-length, s, and the time, t, respectively for a curve.

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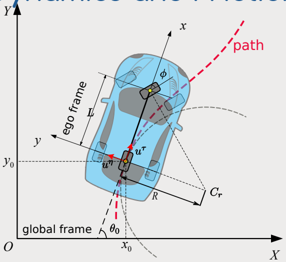
Methods

Dynamics and Friction Circle Constraints

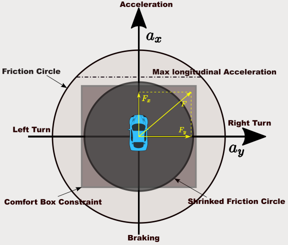
θ	heading of the car
u^{τ}	lateral force in ego frame
u^{η}	longitudinal force in ego frame



Dynamics

$$
\begin{cases}
R\boldsymbol{u} = m\ddot{r}(s), \text{ dynamics equation} \\
\ddot{r}(s) = r''(s)\beta(s) + r'(s)\alpha(s) \\
R = \begin{bmatrix} \cos(\theta(s)) & -\sin(\theta(s)) \\ \sin(\theta(s)) & \cos(\theta(s)) \end{bmatrix}, \text{ rotation matrix} \\
\boldsymbol{u} = (u^{\tau}, u^{\eta}), \text{ control vector}
\end{cases}
$$

Affine!



Friction Circle

$$
\begin{cases}
(\alpha(s), \beta(s), \boldsymbol{u}(s)) \in \Big\{ (\ddot{r}(s), \dot{r}^2(s), \boldsymbol{u}(s)) \mid \\
\qquad \|\boldsymbol{u}(s)\| \le \mu m g, \\
\qquad u^{\tau}(s) \le m \cdot a^{\tau}_{max}, \\
\qquad \beta(s) \le v^2_{max} \Big\}
\end{cases}
$$

Convex!

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Methods

Time Efficiency Objective and Time Window Constraints

Time Efficiency

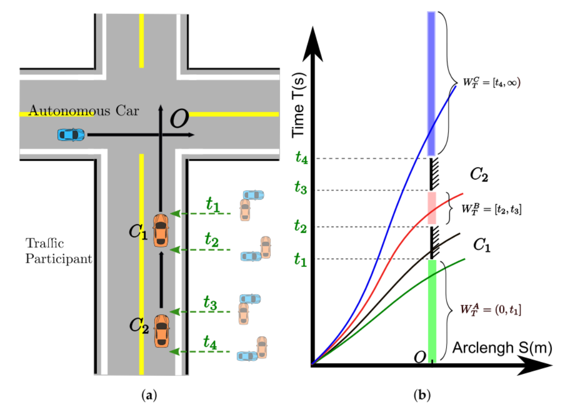
$$J_T = T_{s_f} = \int_0^{s_f} \beta(s)^{-\frac{1}{2}} ds$$

Convex!

Time Window

$$T_i = \int_0^{s_i} \beta(s)^{-\frac{1}{2}} ds \in (0, T_U]$$

Convex!



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Methods

Smoothness Objective

- The jerk $J(s)$ of the speed:

$$\mathcal{J}(s) = \dddot{f} = \dot{\alpha}(s) = \alpha'(s)\dot{f}$$
$$= \alpha'(s)\sqrt{\beta(s)} = \frac{1}{2}\beta''(s)\sqrt{\beta(s)},$$

  which is nonlinear and non-convex.(Common used but not a good option for optimization!)

- The **pseudo jerk**, $\alpha'(s)$:

  the first derivative of acceleration with respect to the arc-length, s.

  The smoothness function is defined as

$$J_S = \int_0^{s_f} \|\alpha'(s)\|^2 ds,$$

  which is **convex**!(Our Choice!)

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Methods

Overall Problem Formulation – Convex Optimization Problem

$$\underset{\alpha(s),\beta(s),u(s)}{\text{minimize}} \quad J = \underset{\text{(Time Efficiency)}}{J_T} + \underset{\text{(Smoothness)}}{\omega J_S}$$

s.t.

(Vehicle Dynamics) $\quad R\boldsymbol{u} = m\ddot{r},$

(State Constraints) $\quad \beta'(s) = 2\alpha(s), s \in [0, s_f],$

(Friction Circle) $\quad (\alpha(s), \beta(s), \boldsymbol{u}(s)) \in \Big\{ (\ddot{r}(s), \dot{r}^2(s), \boldsymbol{u}(s)) \Big| \|\boldsymbol{u}(s)\| \leq \mu m g, \; u^\tau(s) \leq m \cdot a^\tau_{max}, \beta(s) \leq v^2_{max} \Big\},$

(Boundary Condition) $\quad \underline{\alpha}_{s_f} \leq \alpha_{s_f} \leq \bar{\alpha}_{s_f}, \quad \underline{\beta}_{s_f} \leq \beta_{s_f} \leq \bar{\beta}_{s_f},$

(Time Window) $\quad t_i = T(s_i) \in W_T = (0, T_U].$

Red parts are our contributions.

Note that $\alpha(s), \beta(s), u(s)$ are the decision variables. The coefficient $\omega \in \mathbb{R}_+$ is fixed in advance.

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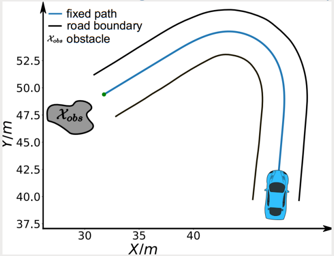
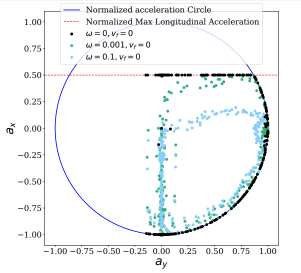
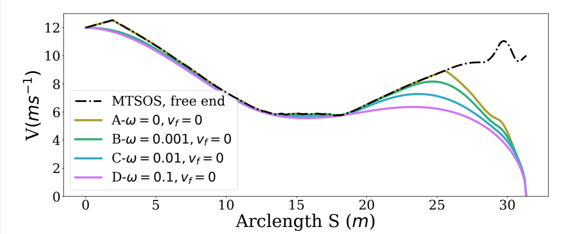
Featured Results

Speed Planning for Safe Stop







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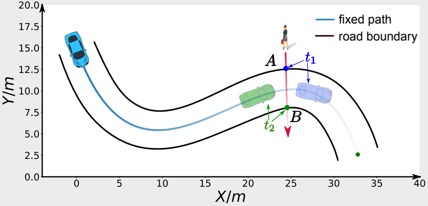
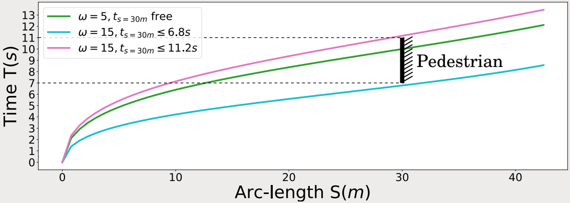
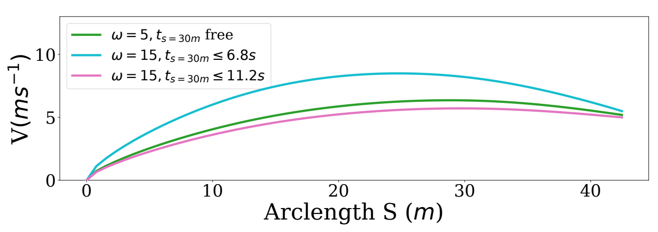
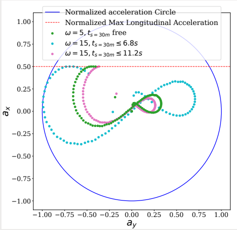
Featured Results

Speed Planning Dealing with Jaywalking on a curvy road









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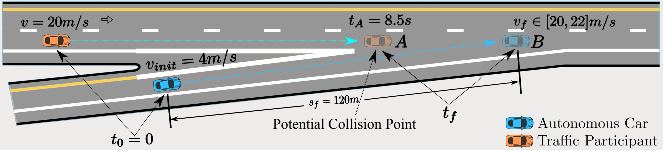
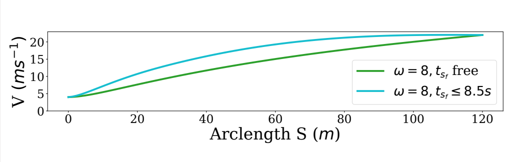
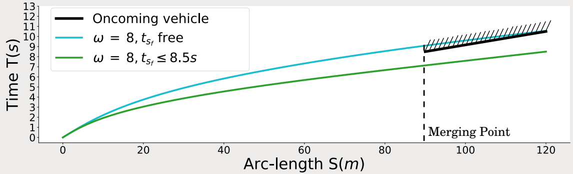
Featured Results

Speed planning for Freeway Entrance Ramp Merging







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Featured Results

Compare to State-of-the-art Methods

Method	S	TE	FC	TW	BC	Optimality	Safety	Mobility	Flexibility
Li *et al.* [1]	✓	✗	✗	✗	✓	✗	low	low	low
Gu *et al.* [2, 3, 4]	✓	✗	✗	✗	✓	✗	medium	medium	medium
Dakibay *et al.* [5]	✗	✗	✓	✗	✓	✗	medium	high	low
Liu *et al.* [6]	✓	✓✗	✗	✓	✓✗	local	medium	medium	medium
Lipp *et al.* [7]	✗	✓	✓	✗	✗	global	low	high	low
Ours	✓	✓	✓	✓	✓	global	high	high	high

S: smoothness, **TE**: time efficiency, **FC**: friction circle, **TW**: time window, **BC**: boundary condition

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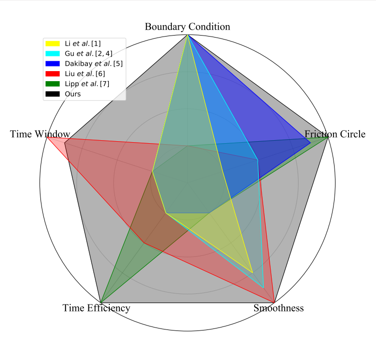
Conclusions

Features and Capacities of Our Method

Capacity: general, flexible, complete, and safe

- maintain the smoothness of the speed profile
- drive within the limits of the friction circle
- consider the time efficiency
- determine the end boundary condition of the state
- perform a precise safe stop
- control the arrival time at a certain point on the path (time window)
- guarantee global optimality (convex optimization)



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Results

Expect more improvements?

Please see our follow-up journal papers:

Zhang Y, Chen H, Waslander S, Yang T, Zhang S, Xiong G, Liu K.

"Toward a More Complete, Flexible, and Safer Speed Planning for Autonomous Driving via Convex Optimization."

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Thank you for your attention!

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